International Journal of Educational Methodology
Volume 10, Issue 2, 279 - 292.
ISSN: 2469-9632
http://www.ijem.com/

Metacognitive Awareness as a Predictor of Mathematical Modeling Competency Among Preservice Elementary Teachers

John Rey G. Oficiar
Central Luzon State University, PHILIPPINES

Edwin D. Ibañez
Central Luzon State University, PHILIPPINES

Jupeth T. Pentang*
Central Luzon State University, PHILIPPINES

Received: January 5, 2024 ▪ Revised: February 28, 2024 ▪ Accepted: March 14, 2024

Abstract: Mathematical modeling offers a promising approach to improving mathematics education. This study aims to determine if the concept of metacognitive awareness in the learning process is associated with mathematical modeling. This study also considers the interaction effect of sex and academic year level on both variables. Focusing the study on preservice elementary teachers might address potential issues and targeted intervention in their preparation program concerning their ability to teach and guide young learners in modeling activities. The research sample includes 140 preservice elementary teachers at Central Luzon State University, Philippines. Data collection used an adapted metacognitive awareness inventory and a validated researcher-made mathematical modeling competency test aligned with the K-12 mathematics curriculum in the Philippines. Results revealed that the preservice elementary teachers had a high metacognitive awareness and mathematical modeling competency, ranging from 22 to 31 out of 36 points. Besides, Factorial ANOVA indicates that academic year level positively affects both variables regardless of sex, and stepwise regression analysis unveiled that information management, declarative knowledge, and planning significantly predict 41.4% of the mathematical modeling competency variance. This suggests that developing metacognitive awareness supports preservice elementary teachers in performing modeling tasks that improve their competency level in mathematics.

Keywords: *Educational research, mathematics education, mathematical modeling, metacognitive processes, teacher preparation.*

To cite this article: Oficiar, J. R. G., Ibañez, E. D., & Pentang, J. T. (2024). Metacognitive awareness as a predictor of mathematical modeling competency among preservice elementary teachers. *International Journal of Educational Methodology*, *10*(2), 279-292. https://doi.org/10.12973/ijem.10.1.1079

Introduction

The Philippine mathematics curriculum aims to nurture critical thinking and problem-solving skills among Filipino learners (Department of Education, 2016). Teachers impart mathematical concepts to solve routine and non-routine mathematical problems to achieve this. In addition, educators embraced the pedagogical approach to teaching mathematics by incorporating real-life applications of mathematical concepts into lessons through mathematical modeling (Asempapa & Sturgill, 2019). This technique improves learners' grasp of abstract mathematical topics while preparing them for practical applications in various professions. By relating mathematics to real-world problems, students better understand the topic and its meaning daily.

In this study, the definition of mathematical modeling was inspired by several studies discussing integrating mathematics into the reality of day-to-day scenarios (Kaiser, 2007; Maaß, 2006; Vorhölter et al., 2019). Mathematical modeling was recognized as a promising approach in mathematics education, as it involves solving real-world problems by translating them into mathematical language (Hidayat et al., 2021). Mathematical modeling competency talks about the ability and willingness of a person to judge, use, and perform mathematical concepts that solve a specific mathematics problem that concerns real-life application (Kaiser, 2007). However, students often face challenges in mathematical modeling tasks, particularly when understanding real-world contexts and making assumptions (Kartal et al., 2017; Mariano-Dolesh et al., 2022).

* **Corresponding author:**
Jupeth T. Pentang, Department of Science Education, Central Luzon State University, Science City of Muñoz, Nueva Ecija, Philippines.
✉ jupeth.pentang@clsu2.edu.ph

© 2024 The author(s); licensee IJEM by RAHPSODE LTD, UK. **Open Access** - This article is distributed under the terms and conditions of the Creative Commons Attribution License (https://creativecommons.org/licenses/by/4.0/).

Learners can build insight into the connection between mathematics and reality by applying their real-life experiences in a mathematics classroom, in which they can apply mathematical concepts by incorporating the use of reality situations (Kaiser, 2007). Due to this, for the learner to understand the real-world context of mathematics, they must be aware and exposed to their environment to understand the connections of mathematical concepts to their everyday life. Hence, if the learners establish this skill at a young age, it will open an opportunity to develop their mathematical skills further.

The development of essential competencies in modeling started from early education, which they already started to develop their competency to learn to model, generalize, and justify at earlier ages. These practices provide students early access to scientific and mathematical reasoning (English & Watters, 2005). Thus, these learners need explicit instructions and proper teacher guidance to develop their essential mathematical modeling competencies. However, it is a challenge for elementary teachers to guide young learners in this complex mathematical task because elementary teachers have few opportunities to implement and integrate mathematical modeling lessons due to the limited practice and training in teacher education programs in their curriculum (Turker & Kaya, 2010). Therefore, the mathematical modeling competency of these professionals must be strengthened.

Problem-solving and mathematical modeling are related concepts in different areas of mathematics education. Mathematical modeling requires competencies, including problem-solving ability (Leong & Tan, 2015). Several studies support that metacognitive awareness affects problem-solving ability (Izzati & Mahmudi, 2018; Mokos & Kafoussi, 2013). Metacognitive awareness was highly encouraged to be incorporated into the problem-solving context of mathematics education to assist educators and learners in the teaching-learning process (Jagals & Van der Walt, 2016). Hence, it could be incorporated into different strategies during the modeling activity (Maaß, 2006). With this, metacognitive awareness was seen as an important influencing factor in developing modeling competencies, as problem-solving strategies are necessary for modeling processes.

For this reason, metacognitive awareness must be strengthened to produce learners with good competency levels in mathematical modeling. The key persons in developing young learners in these concepts are elementary teachers; these professionals are expected to have good knowledge in facilitating self-directed learning. The pedagogical approach to metacognition is pivotal in a learner's metacognitive awareness. Sercenia et al. (2023) and Wilson and Bai (2010) emphasized that this approach deals with how teachers develop the learners to be metacognitively aware, requires them to have an explicit awareness of their metacognition, their ability to control and regulate their thinking process to have a complex understanding of the concept of metacognition, and how to actively guide and facilitate on teaching learners to be metacognitively aware.

However, the development of metacognitive awareness was not explicitly obtained (Cao & Nietfeld, 2007). Desoete and De Craene (2019) pointed out that elementary teachers need exposure to a metacognitive-enriched environment, leading to the deficiency of requisite knowledge about metacognition that hinders them from promoting metacognition in cognitive tasks. This raised the concern that future teachers might need extra training and explicit instructions concerning metacognition during their practices in their preservice education.

Supporting this claim that there is an association between the teachers' use of metacognitive skills during their preservice education and their use of metacognitive skills in their own teaching experiences (Duman & Semerci, 2019). Therefore, determining the current competency level in mathematical modeling, metacognitive awareness, and the association of these concepts among preservice elementary teachers can inform curriculum development that can guide the improvement of targeted training programs to enhance teachers' understanding of mathematical modeling and metacognitive awareness. By focusing on preservice elementary teachers, we can address potential issues at the beginning of their teaching careers, ensuring that they are equipped to guide learners in their thinking process during mathematical modeling task that leads to the development of mathematics education. Hence, assessing the preservice elementary teachers in these concepts is imperative.

This study determined the preservice elementary teachers' current metacognitive awareness and mathematical modeling competency. Moreover, it is essential to note that these concepts might vary on different factors, such as the influence of social role and development factors; addressing this might reduce the potential bias, improving the accuracy of the targeted intervention in teacher education programs and curricula. Specifically, the following questions were answered:

1. What are the preservice elementary teachers' metacognitive awareness levels (in terms of metacognitive knowledge: declarative, procedural, conditional knowledge, and metacognitive regulation: planning, information management, monitoring, debugging, evaluation), and their current competency levels in mathematical modeling?

2. Is there a significant main effect and interaction effect between participants' sex and academic year level on their metacognitive awareness and mathematical modeling competency?

3. Which metacognitive awareness subscales significantly predict the participants' mathematical modeling competency?

Theoretical Framework

Metacognition, rooted in the work of Flavell (1979), which defined it as thinking about thinking, and metacognitive awareness was explored by Schraw and Dennison (1994), which defined as an awareness of an individual's thinking process, and it has two main components; metacognitive knowledge, and metacognitive regulation. Metacognitive knowledge encompasses knowledge about one's cognitive processes and strategies, and it has three critical subscales: declarative knowledge, which deals with knowledge about one's skills, intellectual resources, and abilities as a learner. Procedural knowledge is knowledge about implementing learning procedures; conditional knowledge is when and why to use learning procedures (H. Du Toit, 2017).

Metacognitive regulation involves how individuals regulate their cognitive activities using strategies like planning, which involves setting goals and allocating time and resources to achieve the desired outcome, usually before an activity. Information management requires using various skills and strategy sequences that usually deal with managing information before and during an activity, such as organizing, summarizing, and emphasizing necessary details about the activity, which is essential to help efficiently process information. Monitoring deals with the individual's assessment of their learning or strategy through self-testing and reflection. Debugging involves identifying and correcting errors or problems in an individual's thinking strategies or learning approaches toward the task, and evaluation encompasses analyzing and assessing the effectiveness of strategies and approaches performed during the cognitive activity. Evaluation occurs after learning (H. Du Toit, 2017; Schraw & Dennison, 1994).

Mathematical modeling competency applies mathematical concepts and tools to real-world problem-solving by translating complex situations into mathematical language and making predictions (Tong et al., 2019). Common Core State Standards for Mathematics (CCSSM) presents a structured six-step cycle that includes problem identification, model formulation, computation, interpretation, validation, and reporting (Figure 1).

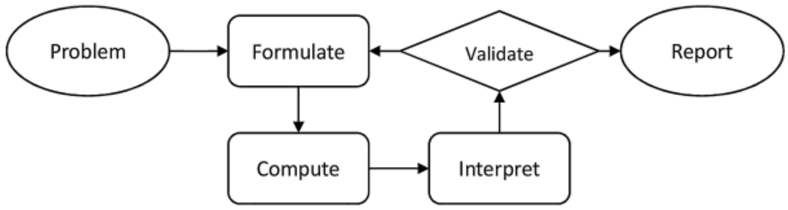

Figure 1. Basic Modeling Cycle for Mathematics by Common Core State Standards (Common Core State Standards Initiative, 2010)

The modeling cycle starts by understanding the concept of the mathematics problem that is based on the real-life scenario; modelers are going to define the relationships of the given and make an assumption, followed by simplifying and structuring the mathematical representations of the defined assumptions on the problem identification to build a mathematical model to represent the problem towards achieving the correct answer. Mathematical methods and concepts must be used to solve the mathematical problem within the model created to obtain the desired result. The results obtained must be interpreted based on the real-life problem. The modeler will validate the answer by linking it to the assumption made; since it is a systematic cycle, if the validation is correct, then the modeler must report the answer comprehensively. Otherwise, the modeler will go back to formulating the problem and adjust accordingly to meet the desired correct answer (Wess et al., 2021).

Self-regulated learning theory, shaped by Zimmerman (2015), plays a pivotal role in the context of these concepts by allowing learners to self-direct their cognitive knowledge into various task-related skills, especially in highly cognitive tasks like mathematical modeling. Metacognitive awareness as self-regulated learning could contribute to performing every sub-competency of mathematical modeling that requires specific learning strategies and approaches from the concept of metacognition, which could significantly improve the domain of mathematical modeling.

As the study proposed, sociodemographic factors, such as sex and academic year level, can influence metacognitive awareness and mathematical modeling competency (Figure 2), linked to differences in thinking, learning styles, and academic background (Steele et al., 2002). These factors impact the awareness levels of metacognitive awareness and modeling skills (Gutierrez de Blume & Montoya, 2023; Mehraein & Gatabi, 2013). The study underlines the importance of considering these effects to enhance the relevance and accuracy of the study, as they contribute to understanding how individual characteristics and sociodemographic factors shape metacognitive awareness and mathematical modeling.

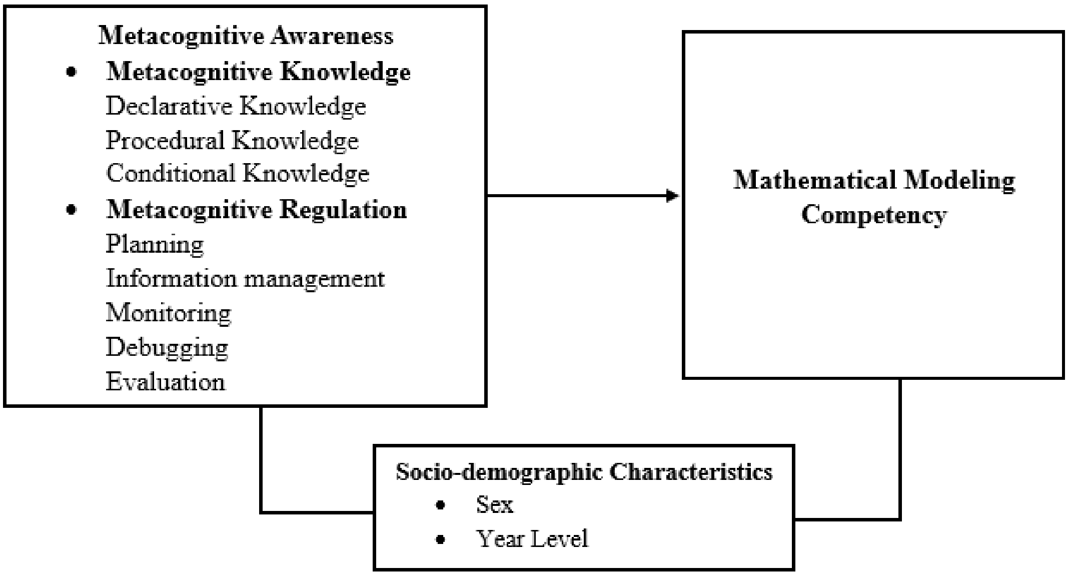

Figure 2. Conceptual Framework of the Study

Literature Review

The relationship between metacognitive awareness and mathematical modeling competency shows a growing interest in mathematics education. In general, metacognitive awareness directly links to the thinking process of an individual, it was seen that it could significantly improve the teaching and learning process. When students face different difficulties, they adjust their thinking process to accommodate the details to elaborate further and better understand the context of the given problem (S. D. Du Toit & Du Toit, 2013).

Metacognition was linked to different areas in mathematics education. Özsoy (2011) found that metacognition was highly correlated to mathematics achievement, and the two components, metacognitive knowledge, and metacognitive skills are significant predictors of mathematics achievement. However, regarding creative ability in mathematical problem solving, Erbas and Bas (2015) indicate that although knowledge of cognition and regulation of cognition is related to creative ability in mathematics, it is not a significant predictor.

Regarding mathematical processes, metacognitive awareness was directly associated with problem-solving as the insightful use of thinking could improve mathematics ability (Schneider & Artelt, 2010). It was emphasized by Izzati and Mahmudi (2018) that students with high metacognition tend to have high problem-solving abilities, which was agreed in the discussion of Mokos and Kafoussi (2013) that the procedural knowledge was dominantly used in solving open-ended mathematical problems applying with specific strategies; information management and debugging strategies were dominant in metacognition that could demonstrate a high level of performance during the task.

While in mathematical modeling, metacognition was seen as an essential factor influencing these processes. The metacognitive knowledge and metacognitive regulation components highly predict mathematical modeling and problem-solving (Arum et al., 2019), aligned with Hidayat et al. (2021), suggesting that individuals with high levels of regulation of cognition can reach a complete problem-solving process than those with medium levels of regulation of cognition. Consequently, individuals who lack metacognitive awareness tend to perform poorer and have difficulties understanding the problem, selecting appropriate strategies, and finding correct answers (Güner & Erbay, 2021).

The sub-dimension of metacognition was explored by Hidayat et al. (2023), who found that the metacognitive strategies in terms of planning and self-checking could positively influence the horizontal mathematization approach, while only the self-checking in cognitive strategy was found to be significant in vertical mathematization approach that affects the modeling proficiency of a learner. Hidayat et al. (2018) emphasized the mediating effect of metacognition dimensions; metacognitive awareness, planning, cognitive strategy, and self-checking significantly mediated the effect of achievement goals in mathematical modeling. Interestingly, Hıdıroğlu and Güzel (2016) tried to explain the transition between cognitive and metacognitive activities in the mathematical modeling process within the technology-enhanced environment, and it was found that cognitive and metacognitive activities did not occur sequentially in the process. Instead, they formed simultaneous and intertwined processes in the modeling process.

Considering educational programs, several researches have been conducted to explore the metacognitive awareness of preservice teachers. The relationship between preservice teachers' beliefs and metacognitive awareness is positively correlated. In contrast, their metacognitive knowledge and metacognitive regulation predict their mathematics teaching that led to the development of improved learning processes in mathematics (Hart & Memnun, 2015), parallel to the study of Alzahrani (2017), which suggests that interventions should directly target the monitoring and regulating of cognition of the learners as it gave notable impact on the perception and thinking process of the learners. It was supported by Louca (2003) that metacognition might aid educational development in effective teaching and learning that produced creative, independent lifelong learners, especially in mathematics education.

However, despite these improvements, the domain of metacognitive awareness has not yet been fully explored. One possible reason suggested by Vorhölter et al. (2019) is a vague concept developed with different conceptualizations. Hence, measuring metacognitive awareness may differ in different contexts, such as when related to highly cognitive tasks like mathematical modeling. Hence, it is imperative to explore the association of these variables further and determine which subscale of metacognitive awareness could significantly affect the mathematical modeling competency. Moreover, diverting the focus on determining these concepts in preservice elementary teachers will contribute to understanding the metacognition processes in mathematical modeling that might address targeted intervention in their preparation in teacher education programs.

Methodology

Research Design

Descriptive design was employed to describe the participants' metacognitive awareness and competency level in mathematical modeling. Besides, a comparative research design was used to determine the main effect and interaction effect between sex and year level on the participants' metacognitive awareness and mathematical modeling competency. Finally, a correlational design was utilized to determine the significant relationship between metacognitive awareness and mathematical modeling competency, specifically to find which subscale of metacognitive awareness significantly predicts the mathematical modeling competency of the participants using regression analysis.

Sample and Data Collection

The research sample consisted of 140 preservice elementary teachers from first year to fourth year taking Bachelor of Elementary Education enrolled in Central Luzon State University in the Philippines, selected using stratified sampling procedure, which allows the researchers to obtain a diverse research sample that represents every group in the population of interest, concerning sex, and the academic year level.

In this study, the 1st- and 2nd-year students were classified as lower-year, and the 3rd-year and 4th-year students as upper-year. Then, grouping them based on the criteria that have been met; to differentiate the two groups based on the expected learning competency and learning experience. The upper-year level already had an opportunity to apply their knowledge and experiences in teaching as they already had the opportunity to teach in the classroom setting. Suggesting that it might significantly impact mathematical knowledge and thinking, focusing research on preservice elementary teachers' metacognitive awareness and mathematical modeling competency is crucial for improving mathematics education and enhancing teaching practices.

Instruments Used

This study's research instrument comprises the Metacognitive Awareness Inventory and the Mathematical Modeling Competency test.

Metacognitive Awareness Inventory. To measure the metacognitive awareness of the participants, the researcher utilized an adapted scale from the modified Metacognitive Awareness Inventory of S. D. Du Toit and Du Toit (2013), initially from the bipolar rating scale of Schraw and Dennison (1994), changed to a five-point Likert scale reflecting the categories (1) strongly disagree, (2) disagree, (3) neutral, (4) agree, and (5) strongly agree. The adapted instrument has high reliability ($\alpha = .94$), while for assessing the two components, metacognitive knowledge ($\alpha = .86$, highly reliable) and metacognitive regulation ($\alpha = .91$, very highly reliable).

Mathematical Modeling Competency test. To assess students' competency in mathematical modeling, the researcher constructed a 3-test item question based on the competency areas of the mathematics curriculum in the Philippines: number sense and algebra, geometry and measurement, and probability (Department of Education, 2016).

An analytical rubric developed by Leong (2012) was modified to objectively score the responses based on the modeling cycle proposed by CCSSM. Each sub-competency of the mathematical modeling was determined: problem, formulate, compute, interpret, validate, and expose, whereas each response in sub-competency was analyzed and rated by the researcher based on the level of competency performed. The indicated level corresponded to the point obtained from the modeling activity. A participant could get 12 points per item, which yields 36 points. Table 1 provides a more detailed scoring guide. A pilot testing of the instrument was conducted, and recommendations from university professors and external parties were noted.

Table 1. Mathematical Modeling Competency Indicators and Scoring Guide.

Sub-Competency	Process	Remarks	Score
Problem	States the problem clearly and can determine the variables	Acceptable and complete	2
		Acceptable but incomplete	1
		Incorrect or no answer	0
Formulate	Creates the model that clearly states all the assumptions that describe the relation of the variables	Comprehensive and relevant to the model	2
		Irrelevant to the model	1
		Incorrect or no answer	0
Compute	Apply heuristic strategies to solve mathematical problem	Computation is clear and accurate	2
		Minor errors are shown in the computation	1
		Incorrect, error in solution	0
Interpret	Refer the results obtained in the model to the real situation and thus achieve real results	Acceptable and complete	2
		Not Clearly Stated	1
		Incorrect or no answer	0
Validate	Check the real results in the situation model for adequacy	Reasoning is logical	2
		Reasoning is somewhat logical	1
		Incorrect or no connection of reason	0
Expose	Summarizes the results that are based on the assumption.	Answers are connected in all stages.	2
		Some answer is not connected	1
		No connection for each stage	0
Total			**/12**

The content of the instrument was clarified to the participants. The participants were given 1 hour for the mathematical modeling competency test that was administered face-to-face, and they needed to answer the adapted metacognitive awareness inventory after the modeling activity. A letter of intent was provided to conduct the study to the target participants before distributing.

Analyzing of Data

This study used various methods of analyzing gathered data, all aligned with the study's stated objectives. Descriptive statistics was used to determine the participants' metacognitive awareness and competency level in mathematical modeling, specifically using the mean percentage score and standard deviation. The computed means for metacognitive awareness were transmuted to qualitative descriptions, which include low level (1.00 to 2.33), average level (2.34 to 3.67), and high level (3.68 to 5.00). Furthermore, to determine the competency level in the mathematical modeling of the participants, a norm-based reference approach was utilized to obtain a reference group that determined the average competency level of the sample using the mean average and the standard deviation (Mean minus *SD* to Mean plus *SD*) and the other groups were categorized and classified as: above average, and below average.

Under inferential statistics, a Factorial Analysis of Variance (Factorial ANOVA) was utilized to determine the main effect and interaction effect between sociodemographic characteristics in terms of sex and year level to the main variables: metacognitive awareness and mathematical modeling competency, and a post-hoc analysis was performed to determine which group of participants performed better. Additionally, assumptions were tested on selecting a regression analysis in determining the possible predictors of mathematical modeling competency; data revealed that variables have a linear relationship, there are no significant outliers, data show independent observation, and the data gathered were homoscedastic. Hence, a stepwise regression analysis was performed to determine which subscale of metacognitive awareness in its two components significantly predicts the mathematical modeling competency.

Results

The preservice elementary teachers are fully aware of their metacognition (Mean = 3.71, *SD* = .416), enabling good self-directed learning by providing practical learning strategies (Table 2). In terms of components, metacognitive knowledge is at an average level (Mean = 3.55, *SD* = .479), consisting of three subscales that are also at an average level: Declarative knowledge (Mean = 3.44, *SD* = .518), Procedural knowledge (Mean = 3.53, *SD* = .513), and conditional knowledge (Mean = 3.67, *SD* = .590).

The metacognitive regulation indicates a high level of control of thinking processes to facilitate learning (Mean = 3.82, *SD* = .423). It includes five subcomponents, which refer to steps that learners must take to regulate and modify the progress of their task. Three subscales were reported at a high level: planning (Mean = 3.88, *SD* = .494), information management (Mean = 3.82, *SD* = .507), and debugging (Mean = 4.16, *SD* = .517), while monitoring (Mean = 3.60, *SD* = .540), and evaluation (Mean = 3.66, *SD* = .491) are at an average level.

Table 2. Level of Metacognitive Awareness in Two Components and Its Subscale

Metacognitive Awareness	Mean	*SD*	Level
Metacognitive Knowledge	3.55	.479	Average
Declarative Knowledge	3.44	.518	Average
Procedural Knowledge	3.53	.513	Average
Conditional Knowledge	3.67	.590	Average
Metacognitive Regulation	3.82	.423	High
Planning	3.88	.494	High
Information management	3.82	.507	High
Monitoring	3.60	.540	Average
Debugging	4.16	.517	High
Evaluation	3.66	.491	Average
Overall Metacognitive Awareness	3.71	.416	High

Note: Low (1.00 to 2.33), Average (2.34 to 3.67), High (3.68 to 5.00)

The participants' mathematical modeling scores had a mean score of 26.83 and a standard deviation of 4.328, providing a basis for categorizing their competency levels using a norm-based approach (Table 3). The competency levels were divided into three categories: below average (20% of participants), average (67.1% of participants), and above average (12.9% of participants). Hence, this indicates that the participants' expected mathematical modeling competency scores are between 22 to 31 points, which is higher by 18 points, the expected half of the total score. Scores below this range are below-average modelers, and those above this range are above-average modelers relative to their group. This norm-based approach offers a standardized framework for comparing individual performance to the reference group, helping researchers assess whether an individual's performance is below or above the norm.

Table 3. Levels of Mathematical Modeling Competency of the Participants

Outcome Variable		Frequency	Percent
Level of Mathematical Modeling Competency	Above Average	18	12.9
	Average	94	67.1
	Below Average	28	20.0
	Total	140	100.00

Dependent Variable: Mathematical Modeling Competency [(Mean = 26.83, *SD* = 4.328), Legend: Low (1 to 21), Average (22 to 31), High (32 to 36) (Norm-based approach)

Results from Factorial ANOVA (Table 4) revealed that sex does not significantly affect metacognitive awareness ($F = 1.547, p > 0.05$), and mathematical modeling competency $F = 30.60, p > .05$). Furthermore, the interaction effect between sex and year level on metacognitive awareness ($F = 3.221, p > .05$) and mathematical modeling competency is insignificant ($F = .323, p > .05$), which indicates that the influence of sex and year level on both concepts are independent of each other implying that both factors are relatively stable and not contingent upon the interaction between them, which suggest focusing on the main effects of the factors on both concepts independently.

Table 4. Main and Interaction Effect of Sociodemographic Characteristics on Metacognitive Awareness and Mathematical Modeling Competency

Sociodemographic Characteristics	Metacognitive Awareness		Mathematical Modeling Competency	
	F	*p*	*F*	*p*
Sex	1.547	.216	30.60	.189
Year Level	6.731	.011	177.69	.002
Sex*Year Level	3.221	.075	0.323	.892

On the other hand, year level does show a significant effect on metacognitive awareness ($F = 6.731, p < .05$) and mathematical modeling competency ($F = 177.69, p < .05$) regardless of sex, which confirmed through post-hoc analysis in Table 5 that upper-year levels exhibit significantly higher metacognitive awareness and better performance in mathematical modeling.

Table 5. Mean Difference between Upper-year Level and Lower-year Level

Academic Year Level	Metacognitive Awareness			Mathematical Modeling Competency		
	Mean	Mean diff. (b-a)	*p*	Mean	Mean Diff. (b-a)	*p*
Lower-year level (a)	3.628	0.186	.011	25.777	2.331	.002
Upper-year level (b)	3.814			28.108		

The stepwise regression analysis indicates that the three subscales from the metacognitive awareness inventory were significant in predicting competency in mathematical modeling (Table 6). The analysis revealed that information management (β = 2.290), declarative knowledge (β = 1.995), and planning (β = 2.162) explain 41.4% of the variance in mathematical modeling competency. These sub-scales are statistically significant ($p < .05$) and predict mathematical modeling competency variance, suggesting that a unit increase in each predictor indicates a specific increase in mathematical modeling competency relative to each predictor's indicated beta score. Suggesting that the equation of regression analysis is $Y = 2.289 + 2.290x_1 + 1.995x_2 + 2.162x_3$ where Y = mathematical modeling competency; x_1 = information management, x_2 = declarative knowledge, and x_3 = planning, which allows to estimate mathematical modeling competency score.

Table 6. Subscales of Metacognitive Awareness That Predicts Mathematical Modeling Competency

Metacognitive Awareness	β	p	R	R^2
Constant	2.289	.000	.643	.414
Information Management	2.290	.003		
Declarative Knowledge	1.995	.006		
Planning	2.162	.007		

Note: Dependent Variable (Mathematical Modeling Competency)

Discussion

Levels of Metacognitive Awareness

Preservice elementary teachers' varying levels of metacognitive awareness are connected to how they facilitate their learning and how they will handle the thinking processes of their future learners. Hence, this reports the current state of preservice elementary teachers and helps us understand their professional learning and development. It significantly affects the development and focuses on the targeted interventions that might need to be improved in their training practices.

This study unveils that preservice elementary teachers exhibit an average knowledge of cognition. Specifically, their understanding of their knowledge or active learning approaches (declarative knowledge), the different techniques and strategies that are effective in the process of doing the tasks (procedural knowledge), and their capabilities to determine why and when they need to apply and implement a specific strategy on a particular task (conditional knowledge).

While participants possess a high level of metacognitive awareness in their regulation of cognitive processes, specifically, they excel in planning, information management, and debugging. The participants have a strong awareness of regulating goal setting and selecting different strategies and approaches before and during cognitive tasks, organizing and effectively managing acquired information in a task, and their ability on error detections and self-corrections leading to an adjustment during cognitive activity. Monitoring and evaluation are average, suggesting that participants have a fair understanding and possess moderate skill in judging the effectiveness and appropriateness of the implemented strategies towards attaining the goal. Moreover, they displayed a strong awareness of their metacognition, allowing them to easily navigate their thinking processes and guide their future learners on different thinking and learning strategies, especially in mathematics education.

Agreeing with the varying results on the level of metacognitive awareness among preservice teachers, these learners with high metacognitive awareness tend to excel in evaluating learning strategies and identifying performance errors (Mishra et al., 2019). On the contrary, teachers' metacognitive awareness remains moderate, suggesting a deficiency in metacognitive development within teaching practices (Abu Bakar & Ismail, 2020). Additionally, preschool teachers often lack sufficient knowledge and awareness of metacognition, hindering their ability to provide constructive feedback for regulating the thinking processes of young learners (Temur et al., 2019). Determining these levels opens an opportunity for curriculum developers and educators on which specific area should be focused and strengthened.

Participants' Levels of Mathematical Modeling Competency

It is important to note that participants were exposed to the same learning environment and curriculum. Hence, this yields that the gathered data were based on their expected learning competency. Therefore, to have flexibility and distinguish the performance of each modeler, it is ideal to assess them relative to the performance of their group using a norm-based reference approach; this categorization was patterned to Henning and Keune (2007). This study indicates that preservice teachers possess adequate knowledge of mathematical modeling. However, in this categorization, the below-average group represents individuals with a basic understanding of modeling but fell short of meeting the criteria for modeling sub-competency compared to the average group.

Consequently, the average group demonstrates the expected proficiency in independent modeling and various sub-competencies, including problem analysis and model construction. Finally, the above-average group exhibits a more advanced level of competency with a solid ability to assess relationships critically, apply models to real-life situations, and create more accurate mathematical models. Similarly, Jensen (2007) affirms that a student who can systematically validate the modeling process has the highest degree of competency in mathematical modeling as they are competent enough to initiate the task and work autonomously than those who can only synthesize and determine the mathematical concepts. This served as a valuable framework for evaluating the mathematical modeling competency of the participants.

Mathematical modeling is concerned with mathematical problem-solving techniques used in real-world applications. As a result, seeing mathematical modeling just as solving mathematical problems may result in a lack of expertise in mathematical modeling. Therefore, preservice elementary teachers must develop their skills in modeling activities and produce their mathematical modeling problems by exposing them to intervention with modeling-based courses on using their critical and creative thinking processes to design effective instruction (Pentang et al., 2023; Sevinc & Lesh, 2018).

Main and Interaction Effect of Sex and Year Level on Both Variables

The effect of sex does not show any significant effect on either variable, suggesting that the level of metacognitive awareness and competency in mathematical modeling of males does not differ in female participants, agreeing with Ludwig and Reit (2013) and Misu and Masi (2017). However, this contradicts the gender differences in both concepts, indicating that female students tend to have better metacognitive awareness (Abdelrahman, 2020; Adiansyah et al., 2021; Yurt, 2022). On the other hand, there is an inconsistent finding relating to mathematical modeling; Zhu (2007) indicates that males tend to have higher competency levels than females, while Yurt (2022) argues that females are better, attributing it to a mediating effect of metacognitive strategies in mathematical reasoning, suggesting that by enabling students to learn and use metacognitive strategies effectively could reduce the gender differences in mathematical reasoning. Hence, this suggests that further research is needed.

Moreover, participants at higher academic year levels exhibit improved metacognitive awareness and mathematical modeling competency regardless of sex, parallel to Van der Stel and Veenman (2010). This indicates that the thinking process and mathematical modeling might improve through successive years of education with the potential impact of exposure to different learning contexts in mathematics education, leading to enhanced metacognitive and mathematical modeling skills (English et al., 2016).

An increased awareness of metacognitive skills plays a crucial role in learning, problem-solving, and mathematical modeling. This improvement may be attributed to students' different learning experiences and cognitive development as they progress through the academic years (Hashempour et al., 2015; Memnun & Akkaya, 2009). These findings are consistent with Hidayat et al. (2018), which suggests that learners at higher academic year levels are more aware of metacognitive strategies and tend to exhibit better mathematical modeling competency, highlighting the importance of metacognition in problem-solving and mathematical modeling.

Predictors of Mathematical Modeling Competency

The study emphasized the importance of metacognitive awareness in mathematical modeling (Chan et al., 2023). Metacognitive awareness involves the understanding of how one approaches and solves mathematical problems. This indicates that the different strategies or approaches during modeling tasks, adept at managing information and planning will likely guide students more effectively in mathematical modeling. This suggests that more than mathematical content knowledge is required for successful mathematical modeling tasks (Jagals & Van der Walt, 2016). This proves that the awareness of an individual concerning their declarative knowledge also involves heuristics strategies to aid in mathematical modeling; they can critically think about what strategy they need to apply and implement during a cognitive task could give the advantage of a deeper approach to solve the problem (English & Watters, 2005; Mokos & Kafoussi, 2013).

Moreover, properly regulating the thinking process could significantly reduce errors, improving modeling competency (Vorhölter et al., 2019). Consequently, properly regulating information management and developing a good planning strategy will result in a better success rate of modeling activity (Hidayat et al., 2023; Livingston, 2003). This might indicate that the failure in mathematical and problem-solving activities is due to poor understanding of the problem and ineffective construction of assumptions based on the givens and the problem (Kartal et al., 2017).

This confirms that metacognitive awareness might improve decision-making, which is necessary in mathematical modeling (Wess et al., 2021). In different sub-competencies in mathematical modeling, the connection of each stage is essential. Thus, analyzing the concept of the problem and how the givens relate to each other before answering the problem will give a clearer view of how to build a mathematical model and solve the problem—indicating that improving decision-making on a problem-solving task leads to success in accurately solving the mathematical problem.

Hence, if the modeler has an awareness of how to deal with and approach properly managing information could come up with a better planning strategy, such as setting an objective and step-by-step procedure that improves decision-making could lessen the hindrance of distractions, bringing a better execution of strategies towards answering the modeling activity that results to the improvement of modeling competency (Maaß, 2006).

Agreed to Güner and Erbay (2021), students with high metacognitive skills are more likely to solve problems correctly by using appropriate strategies to understand the concept of the given problem. The effectiveness of mathematical modeling processes will increase if the learner can monitor and control their learning processes (Alzahrani, 2017; Mokos & Kafoussi, 2013). Thus, metacognition was associated with learning in mathematical modeling (Mbato & Triprihatmini, 2022).

At the same time, students' failure in mathematical modeling can result from neglecting metacognitive sub-scales when preparing for modeling tasks, which assumes that the role of metacognitive strategies like planning and managing information during modeling activity is a vital component in enhancing modeling performances (Hidayat et al., 2018). Moreover, distinguishing between relevant and irrelevant information and making informed assumptions during mathematical modeling activity and with enough awareness of knowledge in proper regulation of cognition through planning and managing information could improve overcoming mental blockages during cognitive activity (Jagals & Van Der Walt, 2016; Sawuwu et al., 2018) Hence, the role of proper planning and information management regulation in optimizing cognitive resources is essential in mathematical modeling. This emphasized that fostering preservice elementary teachers' metacognitive awareness during their training in their teacher education program could significantly improve their learning and teaching strategies, especially in problem-solving and mathematical modeling, leading to the development of early mathematics education.

Conclusion

This study affirms that improving competency in mathematical modeling is not solely about teaching mathematical content but also about equipping teachers with metacognitive skills that empower them to navigate complex teaching scenarios. It also showed that developmental factors notably impact an individual's thinking process and cognitive ability. These ideas could contribute to a more comprehensive and practical approach to teacher professional development.

This calls for the need to empower and improve the teacher education program by providing training and practices that allow these future teachers to develop their metacognitive awareness. The best strategy is to maximize activities that promote self-regulated thinking and learning by allowing learners to create their thoughts and figure out the relationship and association of concepts. Teachers can integrate the use of concepts, graphic organizers, journal writings, answering self-monitoring thinking guides, active questioning, self-explanations of concepts, and with the proper guidance of the teacher on constructing and providing active feedback.

Moreover, teaching mathematical modeling must balance mathematical content and metacognition. It is vital to incorporate the development of the cognitive and affective ability of the learners in mathematical modeling and problem-solving by exposing learners to a different learning context to appreciate the importance and application of mathematics in real-life scenarios while ensuring an inclusive learning environment that fosters equal participation and gender equality in mathematical activities. Teachers must also consider designing strategies that will cater to the developmental factors of an individual, such as exposing them to an increased learning competency as they progress through their academic year level.

The significant predictors identified in the study shed light on the critical role of metacognitive awareness in shaping the mathematical modeling competency of preservice elementary teachers. These insights have practical implications for teacher education programs, offering a pathway to refine and enhance the preparation of future educators for the challenges of teaching mathematical modeling in elementary classrooms.

Recommendations

Teacher preparation programs could incorporate training that explicitly targets information management, declarative knowledge, and planning skills within the context of teaching mathematical modeling. Practical strategies and interventions could be developed to enhance these skills, ultimately preparing preservice elementary teachers to deliver effective and engaging mathematical modeling instruction in elementary classrooms.

While these results are insightful, it opens an avenue for further research. Future studies could delve deeper into the specific mechanisms through which information management, declarative knowledge, and planning impact mathematical modeling competency. Additionally, ongoing research could explore interventions and strategies that enhance these metacognitive skills and improve mathematical modeling competency.

Limitations

It is critical to understand the study's limitations, such as restricted resources for discussing proficiency levels in mathematical modeling, which results in a limited opportunity to elaborate on mathematical modeling principles.

Another factor that influences the study's sample size, generalizability, and potential confounding variables is time restrictions. Discussing these limitations gives legitimacy to the study and guides future research.

Ethics Statements

The study protocols were reviewed and approved by the Central Luzon State University - Ethics Review Committee approved by the Science City of Muñoz, Nueva Ecija, Philippines [ERC Code: 2023-493]. The participants provided their agreement to willingly and voluntarily participate in this study.

Acknowledgments

The authors sincerely thank the Department of Science and Technology - Capacity Building Program in Science and Mathematics Education and the Central Luzon State University for supporting this research.

Conflict of Interest

The authors declare no conflict of interest.

Authorship Contribution Statement

Oficiar: Conceptualization, design, data analysis, and manuscript drafting. Ibañez: Conceptualization, reviewing the manuscript, supervision. Pentang: Design, critical revision of the manuscript, final editing.

References

Abdelrahman, R. M. (2020). Metacognitive awareness and academic motivation and their impact on academic achievement of Ajman University students. *Heliyon, 6*(9), Article e04192. https://doi.org/10.1016/j.heliyon.2020.e04192

Abu Bakar, M. A., & Ismail, N. (2020). Exploring metacognitive regulation and students' interaction in mathematics learning: An analysis of needs to enhance students' mastery. *Humanities & Social Sciences Reviews, 8*(2), 67-74. https://doi.org/10.18510/hssr.2020.82e07

Adiansyah, R., Corebima, A. D., Zubaidah, S., & Rohman, F. (2021). The correlation between metacognitive skills and scientific attitudes towards the retention of male and female students in South Sulawesi, Indonesia. *International Journal of Evaluation and Research in Education, 10*(4), 1272-1281. https://doi.org/10.11591/ijere.v10i4.21597

Alzahrani, K. S. (2017). Metacognition and its role in mathematics learning: An exploration of the perceptions of a teacher and students in a secondary school. *International Electronic Journal of Mathematics Education, 12*(3), 521-537. https://doi.org/10.29333/iejme/629

Arum, R. P., Widjajanti, D. B., & Retnawati, H. (2019). Metacognitive awareness: How it affects mathematical problem-solving process. *Journal of Physics: Conference Series, 1320*, Article 012054. https://doi.org/10.1088/1742-6596/1320/1/012054

Asempapa, R., & Sturgill, D. (2019). Mathematical modelling: Issues and challenges in mathematics education and teaching. *Journal of Mathematics Research, 11*(5), 71-81. https://doi.org/10.5539/jmr.v11n5p71

Cao, L., & Nietfeld, J. L. (2007). College students' metacognitive awareness of difficulties in learning the class content does not automatically lead to adjustment of study strategies. *Australian Journal of Educational & Developmental Psychology, 7,* 31-46. https://bit.ly/3V0k9Jy

Chan, C. T., Leong, K. E., & Zulnaidi, H. (2023). Undergraduate students' attitudes and mathematical reasoning during the pandemic: The mediating role of metacognitive awareness. *European Journal of Mathematics and Science Education, 4*(3), 169-180. https://doi.org/10.12973/ejmse.4.3.169

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* https://bit.ly/3TzwaV2

Department of Education. (2016). *K to 12 curriculum guide: Mathematics (Grade 1 to grade 10).* https://bit.ly/3IwrXLE

Desoete, A., & De Craene, B. (2019). Metacognition and mathematics education: An overview. *ZDM Mathematics Education, 51,* 565-575. https://doi.org/10.1007/s11858-019-01060-w

Duman, B., & Semerci, Ç. (2019). The effect of a metacognition-based instructional practice on the metacognitive awareness of the prospective teachers. *Universal Journal of Educational Research, 7*(3), 720-728. https://doi.org/10.13189/ujer.2019.070311

Du Toit, H. (2017). *The level of metacognitive awareness of preservice mathematics teachers at a higher education institution* [Masters' thesis, University of the Free State, Bloemfontein]. University of Free State, Communities & Collection. http://hdl.handle.net/11660/7567

Du Toit, S. D., & Du Toit, G. F. (2013). Learner metacognition and mathematics achievement during problem-solving in a mathematics classroom. *TD The Journal for Transdisciplinary Research in Southern Africa, 9*(3), 505-518. https://doi.org/10.4102/td.v9i3.194

English, L. D., Ärlebäck, J. B., & Mousoulides, N. (2016). Reflections on progress in mathematical modelling research. In Gutiérrez, Á., Leder, G.C., Boero, P. (Eds.), *The second handbook of research on the psychology of mathematics education* (pp. 383-413). Sense Publishers. https://doi.org/10.1007/978-94-6300-561-6_11

English, L. D., & Watters, J. J. (2005). Mathematical modelling in the early school years. *Mathematics Education Research Journal, 16*(3), 58-79. https://doi.org/10.1007/BF03217401

Erbas, A. K., & Bas, S. (2015). The contribution of personality traits, motivation, academic risk-taking and metacognition to the creative ability in mathematics. *Creativity Research Journal, 27*(4), 299-307. https://doi.org/10.1080/10400419.2015.1087235

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive developmental inquiry. *American Psychologist, 34*(10), 906–911. https://doi.org/10.1037/0003-066X.34.10.906

Güner, P., & Erbay, H. N. (2021). Metacognitive skills and problem-solving. International *Journal of Research in Education and Science, 7*(3), 715-734. https://doi.org/10.46328/ijres.1594

Gutierrez de Blume, A. P., & Montoya, D. (2023). Exploring the relation between metacognition, gender, and personality in Colombian university students. *Psykhe, 32*(2), 1-21. https://doi.org/10.7764/psykhe.2021.30793

Hart, L. C., & Memnun, D. S. (2015). The relationship between preservice elementary mathematics teachers' beliefs and metacognitive awareness. *Journal of Education and Training Studies, 3*(5), 70-77. https://doi.org/10.11114/jets.v3i5.840

Hashempour, M., Ghonsooly, B., & Ghanizadeh, A. (2015). A study of translation students' self-regulation and metacognitive awareness in association with their gender and educational level. *International Journal of Comparative Literature & Translation Studies, 3*(3), 60-69. https://doi.org/10.7575/aiac.ijclts.v.3n.3p.60

Henning, H., & Keune, M. (2007). Levels of modelling competencies. In W. Blum, P. L. Galbraith, H. W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 225-232). Springer. https://doi.org/10.1007/978-0-387-29822-1_23

Hidayat, R., Hermandra, & Ying, S. T. D. (2023). The sub-dimensions of metacognition and their influence on modeling competency. *Humanities and Social Sciences Communications, 10*, Article 763. https://doi.org/10.1057/s41599-023-02290-w

Hidayat, R., Qudratuddarsi, H., Mazlan, N. H., & Mohd Zeki, M. Z. (2021). Evaluation of a test measuring mathematical modelling competency for Indonesian college students. *Journal of Nusantara Studies, 6*(2), 133-155. https://doi.org/10.24200/jonus.vol6iss2pp133-155

Hidayat, R., Zulnaidi, H., & Zamri, S. N. A. S. (2018). Roles of metacognition and achievement goals in mathematical modelling competency: A structural equation modelling analysis. *PLoS ONE, 13*(11), Article e0206211. https://doi.org/10.1371/journal.pone.0206211

Hıdıroğlu, Ç. N., & Güzel, E. (2016). Teknoloji destekli ortamda matematiksel modellemesürecindeki bilişsel ve üst bilişsel eylemler arasındaki geçişler [Transitions between cognitive and metacognitive activities in mathematical modelling process within a technology enhanced environment]. *Necatibey Faculty of Education Electronic Journal of Science & Mathematics Education/ Necatibey Eğitim Fakültesi Elektronik Fen ve Matematik Eğitimi Dergisi, 10*(1), 313-350. https://bit.ly/49TbxZU

Izzati, L. R., & Mahmudi, A. (2018). The influence of metacognition in mathematical problem solving. *Journal of Physics: Conference Series, 1097*, Article 012107. https://doi.org/10.1088/1742-6596/1097/1/012107

Jagals, D., & Van Der Walt, M. (2016). Enabling metacognitive skills for mathematics problem solving: A collective case study of metacognitive reflection and awareness. *African Journal of Research in Mathematics, Science and Technology Education, 20*(2), 154-164. https://doi.org/10.1080/18117295.2016.1192239

Jensen, T. H. (2007). Assessing mathematical modelling competency. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modeling (ICTMA 12): Education, engineering and economics* (pp. 141-148). Woodhead Publishing https://doi.org/10.1533/9780857099419.3.141

Kaiser, G. (2007). Modelling and modelling competencies in school. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling: Education, engineering and economics - ICTMA 12* (pp. 110-119). Elsevier. https://doi.org/10.1533/9780857099419.3.110

Kartal, O., Popovic, G., Morrissey, S., & Holifield, L. (2017). Students' productive struggle through mathematical modeling. *Illinois Mathematics Teacher, 64*(1), 1-9.

Leong, K. E. (2012). Assessment of mathematical modeling. *Journal of Mathematics Education at Teachers College, 3*(1), 61-65. https://doi.org/10.7916/jmetc.v3i1.736

Leong, K. E., & Tan, J. Y. (2015). Assessment of secondary students' mathematical competencies. In *Proceedings of the 7th ICMI-East Asia Regional Conference on Mathematics Education* (pp. 337-345). Universiti Malaya. http://eprints.um.edu.my/id/eprint/13547

Livingston, J. A. (2003). *Metacognition: An overview* (ED474273). ERIC. https://bit.ly/3Tq1BkQ

Louca, E. (2003). The concept and instruction of metacognition. *Teacher Development, 7*(1), 9-30. https://doi.org/10.1080/13664530300200184

Ludwig, M., & Reit, X. R. (2013). Comparative study about gender differences in mathematical modelling. In G. Nagarjuna, A. Jamakhadia, & E. M. Sam (Eds.), *Proceedings of the 5th International Conference to Review Research on Science, Technology and Mathematics Education* (pp. 48-54). Homi Bhabha Centre for Science Education. https://bit.ly/48Lu9K0

Maaß, K. (2006). What are modelling competencies? *ZDM: The International Journal on Mathematics Education, 38*(2), 113-142. https://doi.org/10.1007/BF02655885

Mariano-Dolesh, M. L., Collantes, L. M., Ibañez, E. D., & Pentang, J. T. (2022). Mindset and levels of conceptual understanding in the problem-solving of preservice mathematics teachers in an online learning environment. *International Journal of Learning, Teaching and Educational Research, 21*(6), 18-33. https://doi.org/10.26803/ijlter.21.6.2

Mbato, C. L., & Triprihatmini, V. (2022). Empowering pre-service English teachers' metacognitive awareness in teaching through reflections. *European Journal of Educational Research, 11*(4), 2497-2512. https://doi.org/10.12973/eu-jer.11.4.2497

Mehraein, S., & Gatabi, S. R. (2013). Gender and mathematical modelling competency: Primary students' performance and their attitude. *Procedia - Social and Behavioral Sciences, 128,* 198-203. https://doi.org/10.1016/J.SBSPRO.2014.03.143

Memnun, D., & Akkaya, R. (2009). The levels of metacognitive awareness of primary teacher trainees. *Procedia - Social and Behavioral Sciences, 1*(1), 1919-1923. https://doi.org/10.1016/J.SBSPRO.2009.01.337

Mishra, B. N., Panwar, N., Barjatya, P., Chouhan, D. S., & Mohapatna, S. C. (2019). An age and gender-based analysis of 'metacognitive awareness inventory' (MCAI) among first-year MBBS students from a Central Indian medical college. *The Journal of Community Health Management, 6*(3), 77-81. http://doi.org/10.18231/j.jchm.2019.018

Misu, L., & Masi, L. (2017). Comparison of metacognition awareness of male and female students based on mathematics ability in Department of Mathematics Education of Halu Oleo University. *International Journal of Education and Research, 5*(6), 43-50. https://doi.org/10.33122/ijtmer.v2i3.118

Mokos, E., & Kafoussi, S. (2013). Elementary students' spontaneous metacognitive functions in different types of mathematical problems. *Journal of Research in Mathematics Education, 2*(2), 242-267. https://doi.org/10.4471/redimat.2013.29

Özsoy, G. (2011). An investigation of the relationship between metacognition and mathematics achievement. *Asia Pacific Education Review, 12,* 227-235. https://doi.org/10.1007/s12564-010-9129-6

Pentang, J. T., Caubang, M. G. M., Tidalgo, A. M. L., Morizo, S. B., Bautista, R. M., Viernes, M. D. D., Bucad, M. L., Jr., & Sercenia, J. C. (2023). Demystifying the relationship between confidence and critical thinking in mathematics among preservice teachers in West Philippines. *European Journal of Educational Research, 12*(4), 1743-1754. https://doi.org/10.12973/eu-jer.12.4.1743

Sawuwu, B. Y., Partana, C. F., & Hairida. (2018). Exploration of procedural knowledge of bilingual students in chemical equilibrium problem solving. *EDUCATUM Journal of Science, Mathematics and Technology, 5*(1), 14-23. https://doi.org/10.37134/ejsmt.vol5.1.3.2018

Schneider, W., & Artelt, C. (2010). Metacognition and mathematics education. *ZDM - International Journal on Mathematics Education, 42*(2), 149-161. https://doi.org/10.1007/s11858-010-0240-2

Schraw, G., & Dennison, R. S. (1994). Assessing metacognitive awareness. *Contemporary Educational Psychology, 19*(4), 460-475. https://doi.org/10.1006/CEPS.1994.1033

Sercenia, J. C., Ibañez, E. D., & Pentang, J. T. (2023). Thinking beyond thinking: Junior high school students' metacognitive awareness and conceptual understanding of integers. *Mathematics Teaching-Research Journal, 15*(1), 4-24. https://bit.ly/3PPyCWc

Sevinc, S., & Lesh, R. (2018). Training mathematics teachers for realistic math problems: A case of modeling-based teacher education courses. *ZDM Mathematics Education, 50,* 301-314. https://doi.org/10.1007/s11858-017-0898-9

Steele, C. M., Spencer, S. J., & Aronson, J. (2002). Contending with group image: The psychology of stereotype and social identity threat. *Advances in Experimental Social Psychology*, *34*, 379-440. https://doi.org/10.1016/S0065-2601(02)80009-0

Temur, O. D., Ozsoy, G., Turgut, S., & Kuruyer, H. G. (2019). Metacognitive instructional behaviors of preschool teachers in mathematical activities. *ZDM Mathematics Education, 51*(4), 655-666. https://doi.org/10.1007/s11858-019-01069-1

Tong, D. H., Loc, N. P., Uyen, B. P., & Giang, L. T. (2019). Developing the competency of mathematical modelling: A case study of teaching the cosine and sine theorems. *International Journal of Learning, Teaching and Educational Research, 18*(11), 18-37. https://doi.org/10.26803/ijlter.18.11.2

Turker, B., & Kaya, Y. S. (2010). Preservice teachers' performances at mathematical modeling process and views on mathematical modeling. *Procedia - Social and Behavioral Sciences, 2*(2), 4622-4628. https://doi.org/10.1016/j.sbspro.2010.03.740

Van der Stel, M., & Veenman, M. V. (2010). Development of metacognitive skillfulness: A longitudinal study. *Learning and Individual Differences, 20*(3), 220-224. https://doi.org/10.1016/j.lindif.2009.11.005

Vorhölter, K., Krüger, A., & Wendt, L. (2019). Chapter 2: Metacognition in mathematical modelling - An overview. In S. A. Chamberlin, B. Sriraman (Eds.), *Affect in mathematical modelling* (pp. 29-51). Springer. https://doi.org/10.1007/978-3-030-04432-9_3

Wess, R., Klock, H., Siller, H. S., & Greefrath, G. (2021). Mathematical modelling. In M. S. Biembengut, W. Blum, H. Doerr, P. Galbraith, T. Ikeda, M. Niss, & J. Xie (Eds.), *Measuring professional competence for the teaching of mathematical modelling. International perspectives on the teaching and learning of mathematical modelling* (pp. 3-20). Springer. https://doi.org/10.1007/978-3-030-78071-5_1

Wilson, N. S., & Bai, H. (2010). The relationships and impact of teachers' metacognitive knowledge and pedagogical understandings of metacognition. *Metacognition and Learning, 5*(3), 269-288. https://doi.org/10.1007/s11409-010-9062-4

Yurt, E. (2022). The mediating role of metacognitive strategies in the relationship between gender and mathematical reasoning performance. *Psycho-Educational Research Reviews, 11*(2), 98-120. https://doi.org/10.52963/PERR_Biruni_V11.N2.07

Zhu, Z. (2007). Gender differences in mathematical problem solving patterns: A review of literature. *International Education Journal, 8*(2), 187-203.

Zimmerman, B. J. (2015). Self-regulated learning: Theories, measures, and outcomes. In J. D. Wright (Ed.), *International encyclopedia of the social & behavioral sciences* (2nd ed., pp. 541-546). Elsevier. https://doi.org/10.1016/B978-0-08-097086-8.26060-1